

Algorithm and analytical model to optimize class-based storage of shuttle-based storage and retrieval systems

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Compared to crane-based automatic storage and retrieval systems (AS/RS), shuttle based storage and retrieval systems (SBS/RS) can often achieve higher throughput and energy demand tends to be lower due to less mass movement. As a result, SBS/RS have become widely used in recent years. They can be found in distribution centers across all industries, but can also be found in warehouses for producing companys to supply the production process. A common application is a SBS/RS, which supplies subsequent picking stations via conveyors. The throughput of a SBS/RS can be increased by the storage management policy class-based storage. This policy assigns totes to storage locations based on their frequency of requests. This leads to the definition of zones for classes of totes (or articles). This paper shows how zoning can be improved by an algorithm wich use troughput as optimization criterion.

Keywords: Shuttle, simulation, model, class-based-storage, troughput, storage, retrieval, algorithm, analytical, SBS/RS

1. INTRODUCTION

SBS/RS consists of one or more shuttle carriers, at least one elevator, a rack structure and a control system [1]. Systems with aisle- and tier-captive shuttle carriers are often used for high throughput demands.

Tier-to-tier shuttle carriers cannot change the aisle, but the tiers. In such cases, the shuttle carrier uses the elevator to change to another tier. This can lead to throughput-reducing waiting times [2].

Aisle- and tier-captive SBS/RS use a shuttle carrier for each tier, which cannot leave the tier. The shuttle carrier and the elevator use buffer locations at each tier to store or retrieve totes. As a result, the horizontal and vertical transport is largely decoupled from one another. Accordingly, aisle- and tier-captive SBS/RS can often achieve a higher throughput than tier-to-tier SBS/RS [2].

The throughput of a SBS/RS can be increased by class-based storage. This raises the question of how the totes can be optimally assigned to storage locations due to their frequency of requests. This paper deals with this question.

This paper is structured as follows: Chapter 1 gives an introduction. Chapter 2 contains an explanation of the existing literature on class-based storage for the considered SBS/RS (2.1), the description of the analytical model (2.2) and the simulation model (2.3) as well as the optimization algorithm for class-based storage (2.4). Chapter 2 ends with the results recieved by the applied algorithm (2.5). Chapter 3 contains the summary of the article.

2. MODEL AND ALGORITHM

2.1 Literature

In [2 – 7] class-based storage is used for aisle- and tier-captive and in [8, 9] for tier-to-tier SBS/RS. The papers

show the effect of throughput increase by class-based storage.

In [4] an ant colony clustering algorithm is used to define zones. There are no results mentioned. The SBS/RS considered does not use any buffer locations and thus deviates from aisle- and tier-captive SBS/RS used predominantly in industrial practice. Without buffer locations, the SBS/RS can only achieve a comparatively low throughput, since the elevator and shuttle carrier have to wait for the transfer of totes.

In [5] a part of the algorithm that is described in detail in this paper, is introduced for the first time by way of example, without mentioning results.

[2, 3] and [6 – 9] show to which extent throughput increases through class-based storage is possible. Moreover, in [2] and [6 – 8] principles are described to achieve a favorable definition of zones. A favorable definition of zones can be achieved by matching as close as possible the throughput of elevators and shuttle carriers in the high frequented tiers.

2.2 Analytical Model

In order to be able to calculate the impact of class-based storage on throughput, the assumptions of uniform distribution of storage locations have to be relaxed. Uniform distribution of storage locations implies that each storage location is requested with the same probability (for retrieval or storage requests). [8] describes an analytical model, that allows adjustable distribution of storage locations. The model is based on single-depth tier-to-tier SBS/RS and is expanded in this paper to aisle- and tier-captive SBS/RS.

The definition of zones is formulated for the sides of the aisle with the matrices $Z_{LP1,k,i}$ (LP1 means left side) and $Z_{LP2,k,i}$ (LP2 means right side) whose rows represent tiers (k) and whose columns storage positions (i). i and LP1/LP2 define the storage location.

The zones are numbered in ascending order (1, 2, ..., $\max(z)$).

$$Z_{LP1,k,i} = \begin{pmatrix} Z_{LP1,1,1} & \dots & Z_{LP1,1,i} \\ \dots & \dots & \dots \\ Z_{LP1,k,1} & \dots & Z_{LP1,k,i} \end{pmatrix} \quad (1)$$

$$Z_{LP2,k,i} = \begin{pmatrix} Z_{LP2,1,1} & \dots & Z_{LP2,1,i} \\ \dots & \dots & \dots \\ Z_{LP2,k,1} & \dots & Z_{LP2,k,i} \end{pmatrix} \quad (2)$$

The vector w_z assigns the probability of requests to each zone.

$$w_z = \begin{pmatrix} w_1 \\ \dots \\ w_{\max(z)} \end{pmatrix} \quad (3)$$

The vector l_z assigns the number of associated storage positions to each zone.

$$l_z = \begin{pmatrix} l_1 \\ \dots \\ l_{\max(z)} \end{pmatrix} \quad (4)$$

The probability of requests for a storage location, based on the entire SBS/RS (or the considered subarea thereof, e.g. an aisle), results as follows:

$$w_{i(k,z),x,LP1} = w_{i(k,z),x,LP2} = \frac{w_z}{l_z} \quad (5)$$

$w_{i(k,z),x,LP1}$ is the probability for requests for the storage position i of the tier k , storage location is on the left side. $w_{i(k,z),x,LP2}$ is the probability for requests for the storage position i of the tier k , storage location is on the right side.

The probability for requests from a tier k results as follows:

$$w_{k,y} = \sum_{i=1}^{n_x} (w_{i(k,z),k,LP1} + w_{i(k,z),k,LP2}) \quad (6)$$

The probability for requests for a position i for a shuttle carrier, that is located in tier k , results as follows:

$$w_{i(k),x,LP1} = \frac{w_{i(k,z),x,LP1}}{w_{k,y}} \quad (7)$$

$$w_{i(k),x,LP2} = \frac{w_{i(k,z),x,LP2}}{w_{k,y}} \quad (8)$$

The model based on equations (1) – (8) for tier-to-tier SBS/RS is described in detail in [8]. For aisle- and tier-captive SBS/RS, the model will be extended as follows. The mean travel time of a single travel of the elevator is:

$$t_{EF,y} = \begin{cases} \sum_{k=1}^{n_y} w_{k,y} \left(\frac{|l_{E,y} + (k-1)l_y|}{v_L} + \frac{v_L}{a_L} \right) \text{ for } |l_{E,y} + (k-1)l_y| \geq \frac{(v_L)^2}{a_L} \\ \sum_{k=1}^{n_y} w_{k,y} \left(2 \sqrt{\frac{|l_{E,y} + (k-1)l_y|}{a_L}} \right) \text{ for } |l_{E,y} + (k-1)l_y| \leq \frac{(v_L)^2}{a_L} \end{cases} \quad (9)$$

v_L is the maximum achievable velocity of the elevator, a_L is the acceleration of the elevator. The deceleration is assumed to be identical. $l_{E,A,y}$ is the distance from the input- or output-point (depending on whether storage or retrieval requests are calculated) to tier 1. The value is given a negative sign if the input-point is above tier 1 (otherwise positive). l_y is the distance between the tiers. n_y is the number of tiers. k is a variable to count tiers. Tier $k = 1, 2, \dots, n_y$.

For the mean cycle time of a single-command cycle of the elevator applies:

$$t_{ES,y} = 2(t_{EF,y} + t_{G,y} + t_{P,y}) \quad (10)$$

$t_{P,y}$ is the switching and positioning time that occurs during each braking operation until the elevator stops. $t_{G,y}$ is the time for tote handling of the elevator. The mean throughput of the elevator [tote/h] is therefore:

$$D_{ES,y} = \frac{3600}{t_{ES,y}} \quad (11)$$

The following equations apply to the calculation of a dual-command cycle of the elevator.

The probability that the elevator will remain in the same tier after finishing a storage request is:

$$w_{0,y} = \sum_{k=1}^{n_y} w_{k,y}^2 \quad (12)$$

In the case of a dual-command cycle, the probability $w_{m,y}$ is the probability to travel, after finishing a storage request, to the tier with the next retrieval request. m is the number of changed tiers (eg $m = 1$, one tier has been changed). The following applies:

$$w_{m,b,y} = \sum_{k=1}^{n_y-m} 2w_{k,y}w_{k+m,y} \quad (13)$$

The mean travel time for changing the tiers between storage and retrieval is calculated as follows:

$$t_{L,y} = \begin{cases} \sum_{m=1}^{n_y-1} w_{m,y} \left(\frac{ml_y}{v_L} + \frac{v_L}{a_L} \right) \text{ for } ml_y \geq \frac{(v_L)^2}{a_L} \\ \sum_{m=1}^{n_y-1} w_{m,y} \left(2 \sqrt{\frac{ml_y}{a_L}} \right) \text{ for } ml_y \leq \frac{(v_L)^2}{a_L} \end{cases} \quad (14)$$

If the location of the input-point differs from that of the output-point, another travel time is required to calculate. After finishing the retrieval request a travel to the input-point is required. The following applies:

$$t_{EFA,y} = \begin{cases} \frac{|l_{E,y} - l_{A,y}|}{v_L} + \frac{v_L}{a_L} \text{ für } |l_{E,y} - l_{A,y}| \geq \frac{(v_L)^2}{a_L} \text{ and } l_E \neq l_A \\ 2 \sqrt{\frac{|l_{E,y} - l_{A,y}|}{a_L}} \text{ für } |l_{E,y} - l_{A,y}| \leq \frac{(v_L)^2}{a_L} \text{ and } l_E \neq l_A \\ 0, \text{ otherwise} \end{cases} \quad (15)$$

$l_{E,y}$ is the position of the input-point, $l_{A,y}$ is the position of the output-point (sign selection as described for $l_{E,A,y}$).

The mean travel time of a dual-command cycle is:

$$t_{EFL,y} = t_{EF,E,y} + t_{EFA,y} + t_{L,y} + t_{EFA,y} \quad (16)$$

The expectation value for the occurrence of the switching and positioning times takes into account the positions of the input- and output-point and the probability of the tier remaining.

$$E_{DS} = \begin{cases} (3 - w_{0,y})t_{P,y} \text{ for } l_E = l_A \\ (4 - w_{0,y})t_{P,y} \text{ for } l_E \neq l_A \end{cases} \quad (17)$$

The mean cycle time of a dual-command cycle of the elevator is:

$$t_{DS,y} = t_{EFL,y} + 2t_{G,x} + 2t_{G,y} + E_{DS} \quad (18)$$

The mean throughput of a dual-command cycle of the elevator [tote/h] is:

$$D_{DS,y} = 2 \left(\frac{3600}{t_{DS,y}} \right) \quad (19)$$

For the calculation of the cycle time of a single-command cycle of the shuttle carrier, the following equations apply.

The probability to travel to a position i in the tier k is:

$$W_{i(k),x} = W_{i(k),x,LP1} + W_{i(k),x,LP2} \quad (20)$$

For the mean travel time of the shuttle carrier applies:

$$t_{EF,x(k)} = \begin{cases} \sum_{k=1}^{n_x} W_{i(k),x} \left(\frac{(k-1)L_x}{v_s} + \frac{v_s}{a_s} \right) \text{ for } (k-1)L_x \geq \frac{(v_s)^2}{a_s} \\ \sum_{k=1}^{n_x} W_{i(k),x} \left(2\sqrt{\frac{(k-1)L_x}{a_s}} \right) \text{ for } (k-1)L_x \leq \frac{(v_s)^2}{a_s} \end{cases} \quad (21)$$

The mean cycle time of the shuttle carrier for a single-command cycle in tier k results in:

$$t_{ES,x(k)} = 2(t_{EF,x(k)} + t_{P,x}) + t_{G,x} + t_{G,\uparrow P,x} \quad (22)$$

$t_{G,\uparrow P,x}$ is the time required for tote handling, pick-up from or set-down to a buffer location. $t_{G,x}$ is the time required for tote handling, pick-up from or set-down to a storage location.

The throughput of the shuttle carrier in the tier k results in:

$$D_{ES,x(k)} = \frac{3600}{t_{ES,x(k)}} \quad (23)$$

The mean throughput of all shuttle carriers in one aisle is:

$$D_{ES,x} = \sum_{k=1}^{n_y} D_{ES,x(k)} \quad (24)$$

For a dual-command cycle of the shuttle carrier, the following equations apply. The probability of remaining in the same position, after a storage, is:

$$w_{0,b,x(k)} = \sum_{i=1}^{n_x} W_{i(k),x,LP1} \frac{W_{i(k),x,LP2}}{1-W_{i(k),x,LP1}} + W_{i(k),x,LP2} \frac{W_{i(k),x,LP1}}{1-W_{i(k),x,LP2}} \quad (25)$$

The probability of a change of position is:

$$w_{m,b,x(k)} = \sum_{i=1}^{n_x-m} \left(W_{i(k),x,LP1} \frac{W_{i+m(k),x,LP1} + W_{i+m(k),x,LP2}}{1-W_{i(k),x,LP1}} + W_{i(k),x,LP2} \frac{W_{i+m(k),x,LP1} + W_{i+m(k),x,LP2}}{1-W_{i(k),x,LP2}} + W_{i+m(k),x,LP1} \frac{W_{i(k),x,LP1} + W_{i(k),x,LP2}}{1-W_{i+m(k),x,LP1}} + W_{i+m(k),x,LP2} \frac{W_{i(k),x,LP1} + W_{i(k),x,LP2}}{1-W_{i+m(k),x,LP2}} \right) \quad (26)$$

For the travel time for changing the position applies:

$$t_{L,b,x(k)} = \begin{cases} \sum_{m=1}^{n_x-1} w_{m,b,x(k)} \left(\frac{mL_x}{v_{max}} + \frac{v_{max}}{a} \right) \text{ for } mL_x \geq \frac{(v_{max})^2}{a} \\ \sum_{m=1}^{n_x-1} w_{m,b,x(k)} \left(2\sqrt{\frac{mL_x}{a}} \right) \text{ for } mL_x \leq \frac{(v_{max})^2}{a} \end{cases}$$

The mean travel time for the dual-command cycle in tier k is:

$$t_{EFL,b,x(k)} = 2t_{EF,b,x(k)} + t_{L,b,x(k)} \quad (27)$$

The switching and positioning times coincide with the probability of remaining in the position $w_{0,b,x(k)}$ twice and with the probability of a position change (sum of the probabilities over m positions) $\sum_{m=1}^{n_x-1} w_{m,b,x(k)}$ three times. The following applies:

$$\sum_{m=1}^{n_x-1} w_{m,b,x(k)} = 1 - w_{0,b,x(k)} \quad (28)$$

The expected value of the switching and positioning times in a dual-command cycle results in:

$$E_{DS,x(k)} = 3t_{P,x} \sum_{m=1}^{n_x-1} w_{m,b,x(k)} + 2t_{P,x} w_{0,b,x(k)} = (3 - w_{0,b,x(k)}) t_{P,x} \quad (29)$$

The mean cycle time for a dual-command cycle is:

$$t_{DS,x(k)} = t_{EFL,b,x(k)} + 2t_{G,\uparrow P,x} + 2t_{G,x} + E_{DS,x(k)} \quad (30)$$

The mean throughput of a shuttle carrier [tote/h] in the tier k is:

$$D_{DS,x(k)} = 2 \left(\frac{3600}{t_{DS,x(k)}} \right) \quad (31)$$

The mean throughput achievable by shuttle carriers for one aisle is:

$$D_{DS,x} = \sum_{k=1}^{n_y} D_{DS,x(k)} \quad (32)$$

With this model, cycle time and throughput can be calculated without the consideration of waiting times. The calculation of waiting times is described in [8] for tier-to-tier SBS/RS and is expanded in this paper aisle- and tier captive SBS/RS.

Single-command cycle, elevator and shuttle carrier

The throughput that could be realized by the elevator in the tier k without consideration of a waiting time is:

$$D_{ES,y(k)} = w_{k,y} D_{ES,y} \quad (33)$$

The achievable mean throughput for the elevator $D_{ES,E,w,y}$ and for the shuttle carrier $D_{ES,w,x(k)}$ in the tier k is determined as follows:

$$D_{ES,w,y(k)} = \begin{cases} \min(D_{ES,y(k)}, D_{ES,x(k)}), \text{ one elevator used} \\ \frac{1}{2} \min(2D_{ES,y(k)}, D_{ES,x(k)}), \text{ two elevators used} \end{cases} \quad (34)$$

$$D_{ES,w,x(k)} = \begin{cases} D_{ES,w,y(k)}, \text{ one elevator used} \\ 2D_{ES,w,y(k)}, \text{ two elevators used} \end{cases} \quad (35)$$

In this context, the word "used" distinguishes between actually existing elevators in an aisle and the elevators actively used for the case in question. For example, an aisle may contain two elevators, but one elevator is responsible for storage requests, the other for retrieval requests. Then only the corresponding elevator becomes active when processing only storage or retrieval requests.

The elevator as well as all shuttle carriers in the aisle reach the following mean throughput in total, over all tiers k :

$$D_{ES,w,y} = \sum_{k=1}^{n_y} D_{ES,w,y(k)} \quad (36)$$

$$D_{ES,w,x} = \sum_{k=1}^{n_y} D_{ES,w,x(k)} \quad (37)$$

Dual-command cycle, elevator and shuttle carrier

The throughput, which could be realized by an elevator in the tier k , without consideration of a waiting time, is:

$$D_{DS,y(k)} = w_k D_{DS,y}$$

The achievable mean throughput for the elevator $D_{DS,w,y(k)}$ and for the shuttle carrier $D_{DS,w,x(k)}$ in the tier k is determined as follows:

$$D_{DS,w,y(k)} = \begin{cases} \min(D_{DS,y(k)}, D_{DS,w,x(k)}), \text{ one elevator used} \\ \frac{1}{2} \min(2D_{DS,y(k)}, D_{DS,w,x(k)}), \text{ two elevators used} \\ D_{DS,w,x(k)} = \begin{cases} D_{DS,w,y(k)}, \text{ one elevator used} \\ 2D_{DS,w,y(k)}, \text{ two elevators used} \end{cases} \end{cases} \quad (38)$$

The elevator(s) and the shuttle carriers reach the following mean throughput in total, over all k tiers:

$$D_{DS,w,y} = \sum_{k=1}^{n_y} D_{DS,w,y(k)} \quad (39)$$

$$D_{DS,w,x} = \sum_{k=1}^{n_y} D_{DS,w,x(k)} \quad (40)$$

2.3 Simulation model

The simulation model determines the maximum average throughput for the selected parameter combination. The study relates to an aisle with two shelves with storage locations: left and right from the aisle. In each scenario, it is assumed that requests are available for processing at any time. The SBS/RS is single-deep. The capacity for totes per travel is one for the elevators and the shuttle carriers. Here, one tote corresponds to one article. Each storage location can store one tote. The capacity of the buffer locations is one. In each scenario, two elevators are used per aisle. The simulation model is used to validate the analytical model.

2.4 Algorithm to optimize class-based storage

The algorithm described below iteratively change the definition of zones. After each (valid) iteration, the throughput is determined. After all iterations have been completed, the optimal result for zoning is returned and also associated parameters for system behavior (throughput, cycle time, waiting time). Using this algorithm, the number of zones can be freely defined, up to the number of storage locations. The algorithm returns the result relatively quickly, when using an analytical model.

The algorithm starts with the definition of zone 1 with the tier closest to the midpoint between the input- and output-point. Position i starts with the value one and is incrementally increased. $i = 1$ implies that only the first two storage locations (left and right) in the permitted tiers is assigned to zone 1, $i = 2$ implies that the first and second position with their four storage locations (left and right) in the permitted tiers are assigned to zone 1. The iterations continue until, for the first time, a valid combination is achieved (all required storage locations of zone 1 can be filled). For each valid combination, the achievable throughput is determined. Thereafter, the position i is further increased to the maximum value. In result allowed tiers near the midpoint of the input- and output-point to the position i are assigned to zone 1 and more distant allowed tiers maybe no longer be assigned to zone 1 in the position i , since the required number of storage locations for zone 1 has already been reached. As soon as the position i corresponds to the maximum value, an additional tier for the assignment of zone 1 is released. Position i is reset to 1 and the iterations start again. All other zones are defined one after the other during these iterations, but all tiers and all storage locations are allowed for subsequent zones (except for the storage locations already assigned to the previous zone). Figure 1 shows the algorithm.

Once all iterations have been performed for zone 1, the storage locations for zone 2 will be iteratively assigned in the same pattern. Positions are already reserved for zone 1 (the definition that previously provided the maximum throughput) can't be assigned to zone 2.

Figure 2 illustrates the approach of the algorithm using an example of three zones, six tiers, and six storage locations. The first iteration is shown (zone 1 is dark gray). $f_{k(1)}$ is the number of allowed tiers for the definition of zone 1. $f_{i(1)}$ is the number of allowed storage positions for the definition of zone 1.

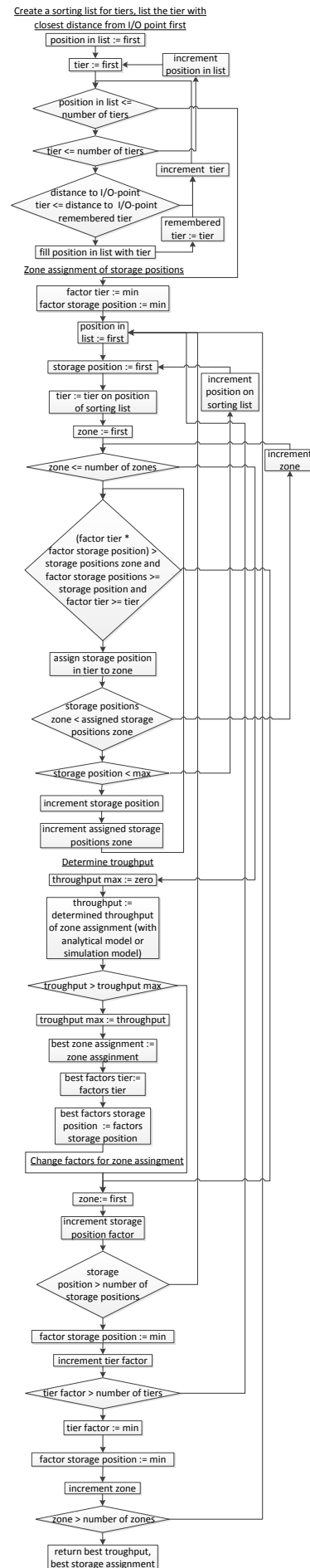


Figure 1: Algorithm

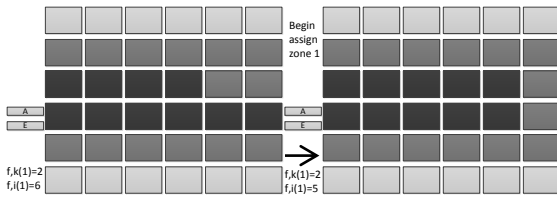


Figure 2: Iterative definition of zones, first iteration

2.5 Results

This chapter shows how to optimize throughput by using the optimization algorithm, and the accordance from the analytical and the simulation model. Table 1 shows the constant parameter values of the investigated SBS/RS.

Table 1: Constant parameter values

Parameter	Value
l_y [m] (Distance between tiers)	0.4
l_{EA} [m] (Distance between first tier and I/O-point)	-1
$n_x (= \max(i))$ (Storage positions per tier)	100
l_x [m] (Distance between storage positions)	0.5
$t_{P,y}$ [s] (Positioning and switching times Elevator)	0.5
$t_{G,x} (= t_{G,\cup P,x})$ [s] (Tote handling time shuttle-carrier)	3
$t_{P,x}$ [s] (Positioning and switching times shuttle-carrier)	0.5
SBS/RS	Aisle- and tier-captive

Table 2 shows the variable parameter values (variants). Table 3 shows the parameter values for different zones. Tables 4 - 10 show the results of the investigated variants. The cursively written values were determined by the simulation model, the others were calculated by the analytical model. Zones = 1 implies random storage assignment. Variants 1 - 3 show the definition of zones and the potential of optimization for different values of acceleration and velocity for elevators and shuttle carriers. Variant 4 shows the possibility of taking account of a lower storage ratio by a zone that is not requested (variant 4 deviates from variant 1 only with a lower storage ratio). Variation 5 shows the effect of zoning for a higher SBS/RS (variant 5 deviates from variant 2 only with 36 tiers). The throughput results in the following tables refer to the throughput of one elevator. In the scenarios shown here, two elevators are used per aisle. Thus, twice the throughput can be achieved per aisle.

Table 2. Variable parameter values

Variant	1	2	3	4	5
n_y (Tiers)	12				36
v_L (Velocity elevator)	1	4	7	1	4
a_L (Acceleration elevator)	2	4	7	2	4
$t_{P,y}$ (Pick-up and set-down time elevator)	1	4	3	1	4
v_S (Velocity shuttle carrier)	5	3.5	2	5	3.5
a_S (Acceleration shuttle carrier)	5	3.5	2	5	3.5
Storage ratio [%]	95			50	95

Table 3. Parameter values for different zones

Zones	1	2	3	4	6
l_z (Storage positions per zone)	$l_z = (1200)$	$l_z = \begin{pmatrix} 400 \\ 800 \end{pmatrix}$	$l_z = \begin{pmatrix} 240 \\ 360 \\ 600 \end{pmatrix}$	$l_z = \begin{pmatrix} 120 \\ 180 \\ 300 \\ 600 \end{pmatrix}$	$l_z = \begin{pmatrix} 100 \\ 140 \\ 160 \\ 200 \\ 280 \\ 320 \end{pmatrix}$
w_z (Probability of request per zone)	$w_z = (1)$	$w_z = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$	$w_z = \begin{pmatrix} 0.6 \\ 0.3 \\ 0.1 \end{pmatrix}$	$w_z = \begin{pmatrix} 0.6 \\ 0.3 \\ 0.1 \\ 0 \end{pmatrix}$	$w_z = \begin{pmatrix} 0.3 \\ 0.25 \\ 0.2 \\ 0.15 \\ 0.06 \\ 0.04 \end{pmatrix}$

Table 4: Results variant 1

		Throughput elevator			
Cycle	Single-command				
Zones	1	2	3	6	
Variant 1	494.01 (494.75)	565.11 (552.42)	632.19 (616.23)	627.98 (615.84)	

Table 5: Results variant 2, single-command cycle

		Throughput elevator			
Cycle	Single-command				
Zones	1	2	3	6	
Variant 2	319.34 (319.7)	331.71 (332.67)	343.83 (341.52)	344.14 (341.14)	

Table 6: Results variant 2, dual-command cycle

Variant 2	Throughput elevator				
Cycle	Dual-command				
Zones	1	2	3	6	
Variant 2	344.89 (344.08)	353.9 (350.93)	365.64 (358.15)	365.89 (357.75)	

Table 7: Results variant 3, single-command cycle

		Throughput elevator			
Cycle	Single-command				
Zones	1	2	3	6	
Variant 3	466.41 (466.59)	482.46 (480.49)	491.64 (490.82)	487.08 (485.44)	

Table 8: Results variant 3, dual-command cycle

		Throughput elevator			
Cycle	Dual-command				
Zones	1	2	3	6	
Variant 3	512.75 (510.84)	526.65 (522.21)	539.86 (532.53)	528.94 (524.25)	

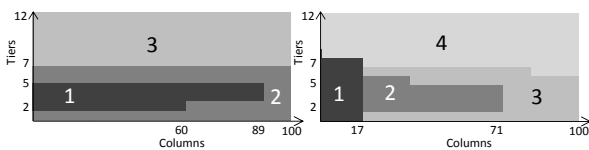
Table 9: Results variant 4

		Throughput Elevator	
Cycle	Single-command		
Zones	1	4	
Variant 4	494.01 (494.75)	659.40 (645.32)	

Table 10: Results variant 5

Cycle	Throughput Elevator			
	Single-command		Dual-command	
Zones	1	3	1	3
Variant 5	259.16 (260.64)	304.41 (303.64)	294.10 (297.98)	328.24 (327.22)

The optimal definition of zones at increased acceleration and velocity of the elevator tends to increase the number of frequently used tiers. The optimal definition of zones at increased acceleration and velocity of the shuttle carriers tends to decrease the number of frequently used tiers, see also [3]. Figure 3 shows two examples of defined zones by the algorithm.

**Figure 3: Optimal zones defined by algorithm, left variant 3, right variant 4**

The results show the potential of optimization for defined zones. Also it can be shown, that the analytical model has only slight deviations from the simulation model for the calculated variants (highest deviation is 2.52 %).

All of the following mentioned percentage increases in throughput refer to the comparison between the zone definition of a variant and random storage assignment. For variant 1, the throughput can be increased by 27.97%, for variants 2 - 3 between 5.29% and 7.77%. The slower the elevator moves (and the faster the tote handling time), the stronger the effect of reducing the path is due to zone formation. Therefore the throughput in variant 1 can be increased significantly more as for variant 2 and 3.

The consideration of a lower storage ratio results in a significant increase in throughput of 33.48% (variant 4).

The farther the elevator has to travel, the higher the potential for optimization. Variation 5 increases throughput by defined zones up to 17.46%. This is a much higher increase than with variant 2, which has the same parameter values as variant 5 with the exception of the number of tiers (variant 5 has 36 tiers, variant 2 has 12 tiers).

3. CONCLUSION

In this paper, an analytical model for calculating the throughput of aisle- and tier-captive SBS/RS is presented. The model allows the consideration of any probabilities of requested storage locations. Hence, modelling the storage management policy class-based storage is possible. The analytical model has a high accordance with the simulation model for the calculated and simulated variants.

Furthermore, an algorithm was presented, that optimize the definition of zones. The algorithm optimize according to the criterion of maximum throughput. The results show the potential of optimization by the application of the algorithm.

Further interesting topics for future work may be:

- Finding optimal zones with evolutionary algorithms (is currently being researched at Heilbronn University).
- Methods of artificial intelligence, e.g. for deep reinforcement learning to solve control and optimization problems (is currently being researched at the Institute for Mechanical Handling and Logistics at the University of Stuttgart).

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